Geometric realisation of Connes spectral triples for algebras with central bases

Evelyn Lira-Torres[1]
Joint work with Prof. Shahn Majid (arXiv:2208.07821)
Queen Mary University of London

WINGs Spring Retreat, Chester eld
18th April 2023

[1]Funded by CONACyT and Alberto y Dolores Andrade Mexico

Outline

NCRG statement for quantum gravity

1. We will not assume that the spacetime is continuum at the plank scale.
2. Instead propose that it is more e ectively described by a noncommutative coordinate algebra.
3. And in the limit classical RG is recovered.

[1] E.J. Beggs and S. Majid, Quantum Riemannian Geometry, Grundlehren der mathematischen Wissenschaften, Vol. 355, Springer (2020) 809pp

Basic NCRG formalism

We work with A a unital algebra, typically a -algebra over C, in the role of `coordinate algebra'.

1. Di erentials are formally introduced as a bimodule ^{1} of 1-forms equipped with a map d : A ! ^{1} obeying the Leibniz rule d(ab) = (da)b + adb.
2. Assume this extends to an exterior algebra (; d) with $d^2 = 0$ and d obeying the graded-Leibniz rule and ^(g) = 0.
3. A quantum metric is g 2 ^{1} $_A$ ^{1} and a bimodule map inverse (;) : ^{1} $_A$ ^{1} ! A.
4. A bimodule connection on ^{1} is r : ^{1} ! ^{1} $_A$ ^{1} obeying:
   1. r (a:!) = a:r ! + da !,
   2. r (!: a) = (r !):a + (! da)

   with a unique `generalised braiding' bimodule map : ^{1} $_A$ ^{1} ! ^{1} $_A$ ^{1}.

General axiomatic motivation

In recent years, the quantum Riemannian geometry was extended to a systematic theory including the QLC and further structure as:

1. 'spinor' bimodule S equipped with a bimodule connection ∇_S
2. A 'Clifford action' $\triangleright : \Omega^1 \otimes_A S \to S$.
3. Leading to a quantum-geometric Dirac operator $D = \triangleright \circ \nabla_S$.
4. And a inner product used to complete S to a Hilbert Space.

Motivation for central bases

We say a basis e^i is central if is a grassmann algebra: $e^i e^j + e^j e^i = 0$.

We will think in central bases if A has trivial centre, ^{1} has a central basis $\{s^i\}$ and S has a central basis $\{e^\alpha\}$.

The central bases assumtion resumes into the set of 1-forms are se adjoint. The metric translates to the matrix g_{ij} of metric coe cient in the basis being hermitian. If is the ip map then the -preserving condition on translates to the Christo el symbols in the basis being real.

Geometric realisation of Connes spectral

formalis9051 0.9196 0.9451 rg 0.9051 0.9196 0.9451 RG 0.39262 0.48549 0.64862 rg 0.39262 0.48549 0.64862 RG 0 -7.389 Td158 rg 0 0 04 0 0 8r2ompG /F5 0stric

Some results and examples

Theorem

Up to a phase in the 2D spinor bundle case, J can be obtained with $r > 0$ and $z \in \mathbb{C}$ as either:

$$(1): \quad J = \begin{pmatrix} z & r \\ \frac{\epsilon - |z|^2}{r} & -z \end{pmatrix}; \quad (2): \quad J = \begin{pmatrix} 1 & \frac{\epsilon - 1}{|z|^2} z \\ z & -\frac{z}{z} \end{pmatrix}$$

or its transpose. The $\epsilon = -1$ case of (2) needs $z \neq 0$ and up to a

Some results and examples

Noncommutative Torus

the only geometrically realised Dirac operator for the standard Euclidean metric and a WQLC are

$$D(\ _\alpha e^\alpha) = (\ @\ _\alpha s^i).e^\alpha + \ _\alpha d_i s^i.e^\alpha = \ ^{i\,\alpha}{}_\beta((@+d_i)\ _\alpha)e^\beta: \quad (1)$$

. With Hilbert space, the state $\int u^m v^n = \ _{m;0}\ _{n;0}$